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SHIELDING OF SURFACES IN COUETTE FLOW AGAINST RADIATION BY TRANSPIRATION OF AN ABSORBING–EMITTING GAS

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Abstract—The laminar Couette flow has been analysed to show the effects of the injection of a radiation absorbing–emitting gas on the interaction of convection with radiation, on temperature profiles, and on heat transfer to the stationary surface. Both a constant and a variable physical property cases are considered. Heat-transfer results are presented and it shown that the reduction in radiation heat-transfer rate at the surface by injection of a radiating gas is accompanied by an increase in convective heat-transfer rate. It is found that the effectiveness of the radiating gas in shielding the surface from an incident radiation flux diminishes with decreasing emissivity. The emissivity condition between the black and perfectly reflecting surface extremes above which it is advantageous to inject an absorbing gas is determined.

NOMENCLATURE

- Bo, Boltzmann number, $\sigma T_1^3/mc_{pi}$;
- c_{pi}, specific heat at constant pressure;
- D, binary diffusion coefficient;
- E, Eckert number, $U_1^2/c_{pi}(T_0 - T_1)$;
- E_b, black-body emissive power, σT^4;
- E_n, exponential integral function, $E_n(\tau) = \int_0^1 \mu^{n-2} \exp(-\tau/\mu)\, d\mu$;
- e, local energy flux defined by equation (5);
- e^*, dimensionless local energy flux, $e/(k_1 T_1/\delta)$;
- $\mathscr{F}$, local radiation flux defined by equation (6);
- $\mathscr{F}^*$, dimensionless local radiation flux, $\mathscr{F}/(k_1 T_1/\delta)$;
- F_d, diffuse component of the incident radiation flux;
- F_d^*, dimensionless incident radiation flux, $F_d/\sigma T_1^4$;
- F_p, parallel component of the incident radiation flux;
- F_p^*, dimensionless incident radiation flux, $F_p/\sigma T_1^4$;
- J_0, radiosity defined by equation (7);
- k, thermal conductivity;
- m, mass flux of the injected gas, ρv;
- N, dimensionless conduction–radiation interaction parameter, $k_1\kappa_1/4\sigma T_1^3$;
- Pe, Peclet number, $Re\ Pr = m\delta c_{pi_1}/k_1$;
- Pr, Prandtl number, $\mu_1 c_{pi_1}/k_1$
- q, heat flux to the wall defined by equation (31);
- q^*, dimensionless heat flux to the wall, $q/(k_1 T_1/\delta)$;
- Re, blowing Reynolds number, $m\delta/\mu_1$;
- Sc, Schmidt number, $\mu_1/\rho_1 D_1$;
- T, absolute temperature;
- t, dummy integration variable;
- u, velocity component in the x-direction;
- v, velocity component in the y-direction;
- w, mass fraction of the injected gas;
- y, distance normal to the surface.

Greek symbols

- α, β, γ, parameters in equation (25);
- δ, spacing between plates;
- ϵ, emissivity;
- ϵ_0^*, break even emissivity;
- η, independent variable, τ/τ_0;
- θ, dimensionless temperature, T/T_1;

κ, absorption coefficient;
μ, dynamic viscosity;
μ_o, $\cos \theta_0$;
ξ, dimensionless distance, y/δ;
ρ, density;
σ, Stefan–Boltzmann constant;
τ, optical depth defined by equation (8);
τ_o, optical thickness defined by equation (8);
τ_1, optical distance, $\kappa_1 \delta$;
τ_w, shear stress at the wall;
τ_w^*, dimensionless shear stress at the wall, $\tau_w/(\mu_1 U_1/\delta)$;
Φ, dimensionless radiation flux, $\mathscr{F}/\sigma T_1^4$;
ϕ, dimensionless velocity, u/U_1;
ω, dimensionless mass fraction of injected gas, w/w_0;
χ_0, dimensionless radiosity, $J_0/\sigma T_1^4$.

Subscripts
c, convective;
i, injected gas;
n-r, refers to a non-radiating gas;
0, wall at $y = 0$;
1, wall at $y = \delta$;
w, refers to the wall at $y = 0$.

Superscripts
$+$, refers to dimensionless property, i.e. $k^+ = k/k_1$, etc.

INTRODUCTION

THE PROBLEM of shielding the surface of a space vehicle from radiation emitted by the shock layer during entry into the planetary atmospheres at high speeds is important. Protection of bodies from high intensity thermal radiation emitted from a fireball produced by an atomic explosion is also of considerable practical interest. Gaseous core nuclear reactors, arcs and many others are some of the problem areas in applied physics and engineering where protection of surfaces from incident radiation is required.

It is the purpose of the present paper to determine the effectiveness of mass transfer cooling in shielding a surface from incident radiation by transpiration of an absorbing–emitting gas. Real surfaces are neither perfectly absorbing nor completely reflecting, and the optical thickness of the radiating gas cannot be made infinitely large. Therefore it is of interest to predict the reflectivity of the surface, the flow parameters and the radiation characteristics of the gas which would be able to shield effectively the surface from the incident radiation. Since the geometric complexities introduced by the dependence of the radiation flux on the flow geometry are avoided by considering Couette flow, this simple but physically meaningful flow model was selected for the system. The usefulness of Couette flow has been proven of value in the past for gaining understanding of more complex flow systems since it is a one-dimensional representation of the two dimensional boundary layer.

The effect of radiation absorbing foreign gas on the heat transfer in laminar compressible boundary-layer flow over a blunt body has been investigated by Howe [1] and by Rumynskii [2] for flow over a flat plate. Radiation transfer was assumed to be one-dimensional. Emission of radiation from the gas was neglected in [1] and the gas was treated as transparent in [2]. They have shown that under certain conditions a net saving in total heat transfer (convective plus radiative) can be achieved. Very recently, Fritch *et al.* [3] studied the shielding of a surface from parallel beams of thermal radiation by distributed injection of a fluid containing absorbing particles into an incompressible laminar boundary-layer flow over a gray surface near the stagnation point. Novotny *et al.* [4] has investigated mass transfer cooling of a black surface in high-speed Couette flow of a radiating gas. They have found that radiation flux is relatively unaffected by the rate of mass transfer for small and intermediate values of optical thickness but very influenced by the optical properties of the fluid. Convective heat transfer, on the other hand, is strongly affected by the mass-transfer parameter and relatively insensitive to the optical properties. Viskanta [5] has reported that a stagnant layer of radiating gas can shield a surface effectively from incident radiation if the optical thickness is large, but that the protection diminishes to zero as the surface reflectivity is increased to unity. Heat transfer in Couette flow of a viscous radiating gas without mass transfer cooling has been studied by Greif [6] and Viskanta [7].

In the present paper the problem is formulated exactly and both the constant and variable physical property cases are studied. Due to the large number of independent parameters involved, only selective and representative solutions are given. Results are given for the Couette flow problem having a cool stationary wall as a thermal boundary condition. Situations approximating the low and high-speed boundary-layer flow are examined. The dimensionless parameters governing the interaction of radiation with other heat-transfer processes are varied over a wide range of values of physical interest. The effect of varying the blowing rate and changing the surface emissivity on temperature distribution and heat transfer is established and optimum characteristics determined.

ANALYSIS

Physical model and assumptions

The Couette flow model is illustrated schematically in Fig. 1. The flow is produced by a steady relative motion of two infinite parallel plates. The lower wall is stationary and the upper wall moves in its own plane with a constant velocity U_1. An absorbing–emitting gas is injected uniformly into the main stream from the stationary plate. The other plate then must also be a porous one so that mass as well as heat may pass readily through it to keep the entire system in a steady state. The walls are considered to be isothermal. The stationary wall is assumed to be a gray, diffuse emitter and reflector, and the moving wall is completely transparent to radiation. The injected gas is considered to have an index of refraction of unity and to be gray, i.e. the absorption coefficient is independent of wavelength but varies with temperature and concentration. There is a radiation flux incident on the transparent moving plate from some external source. This radiation flux has a diffuse component F_d

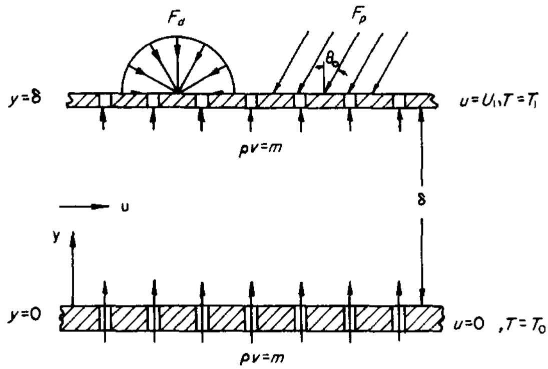

FIG. 1. Physical model and coordinate system.

and a collimated component F_p making an angle θ_0 with the normal to the plates.

Basic equations

The conservation equations which govern Couette flow of non-radiating gas are well known and the development of these equations can be found in [8]. The formulation of the radiation transfer problem is derived in some detail in [6] and need not be repeated here. Thus, the basic conservation equations can be written as:

mass,

$$\frac{\mathrm{d}(\rho v)}{\mathrm{d}y} = 0 \qquad \text{or } \rho v = m = \text{constant}; \tag{1}$$

species,

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(-\rho D\frac{\mathrm{d}w}{\mathrm{d}y} + \rho v w\right) = 0; \tag{2}$$

momentum,

$$\frac{\mathrm{d}}{\mathrm{d}y}\left(\mu\frac{\mathrm{d}u}{\mathrm{d}y} - \rho v u\right) = 0; \tag{3}$$

energy,

$$-\frac{\mathrm{d}e}{\mathrm{d}y} + \mu\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 = 0; \tag{4}$$

where the local energy flux in the y-direction is

$$e = -k\frac{dT}{dy} + mc_{pi}T + \mathscr{F}. \quad (5)$$

The local radiation flux $\mathscr{F}$ can be expressed as

$$\mathscr{F}(\tau) = 2[J_0E_3(\tau) - F_dE_3(\tau_o - \tau) - \tfrac{1}{2}\mu_oF_p e^{-(\tau_o-\tau)/\mu_o} + \int_0^\tau E_b(t)\,E_2(\tau - t)\,dt - \int_\tau^{\tau_0} E_b(t)\,E_2(t - \tau)\,dt]. \quad (6)$$

where the radiosity J_0, i.e. the energy emitted from the stationary wall plus the fraction of incident energy reflected from the wall, is given by:

$$J_0 = \epsilon_0E_{b0} + 2(1 - \epsilon_0)[F_dE_3(\tau_o) + \tfrac{1}{2}\mu_oF_p e^{-\tau_o/\mu_o} + \int_0^{\tau_o} E_b(t)\,E_2(t)\,dt]. \quad (7)$$

In equations (6) and (7) the optical depth, τ and the optical thickness τ_o are defined as:

$$\tau = \int_0^y \kappa\,dy \quad \text{and} \quad \tau_o = \int_0^\delta \kappa\,dy. \quad (8)$$

The boundary conditions for equations (2–4) are assumed to be:

$$u = 0,\ w = w_0,\ T = T_0 \quad \text{at} \quad y = 0, \text{and} \quad (9)$$

$$u = U_1,\ w = w_1,\ T = T_1 \quad \text{at} \quad y = \delta. \quad (10)$$

The momentum (3) and the energy equation (4) can be readily integrated once. Integrating equation (3) with respect to y and using equation (9), we may write:

$$\mu\frac{du}{dy} = mu + \tau_w, \quad (11)$$

where $\tau_w = (\mu du/dy)_{y=0}$ is the shear stress at the stationary surface, $y = 0$. It follows at once that the energy equation can be integrated to give:

$$e = e_w + (m/2)\,u^2 + \tau_w u, \quad (12)$$

if equation (11) is substituted into equation (4). Here e_w is the value of e at $y = 0$. Integration of equation (2) yields:

$$\rho D\frac{dw}{dy}\ \rho v(1 - w) = 0, \quad (13)$$

where the integration constant was evaluated from the condition at the wall, $y = 0$:

$$\left(-\rho D\frac{dw}{dy} + \rho v w\right)_{y=0} = \rho v = m. \quad (14)$$

Introducing dimensionless variables and appropriate parameters, the momentum, and energy equations can be written as:

$$\mu^+\frac{d\phi}{d\xi} = Re\,\phi + \tau_w^* \quad (15)$$

and

$$-(k^+/Pe)\frac{d\theta}{d\xi} + c_p^+\,\theta + Bo\Phi = e_w^* + E(\theta_0 - 1)(\tfrac{1}{2}\phi + \tau_w^*/Re)\,\phi \quad (16)$$

If equations (6) and (7) are combined, the local radiation flux in dimensionless form, Φ, can be expressed as:

$$\Phi(\tau) = 2\{\epsilon_0\theta_0^4E_3(\tau) - [E_3(\tau_o - \tau) - 2(1 - \epsilon_0)\,E_3(\tau_o)\,E_3(\tau)]\,F_d^* - [\tfrac{1}{2}e^{-(\tau_o-\tau)/\mu_o} - (1 - \epsilon_0)\,e^{-\tau_o/\mu_o}E_3(\tau)]\,\mu_0F_p^* + \int_0^{\tau_o} \theta^4(t)\,[\text{sign}\,(\tau - t)\,E_2(|\tau - t|) + 2(1 - \epsilon_0)\,E_3(\tau)\,E_2(t)]\,dt\}. \quad (17)$$

where sign $(\tau - t) = +1$ for $(\tau - t) > 0$ and sign $(\tau - t) = -1$ for $(\tau - t) < 0$. In terms of dimensionless variables, the species equations is given by:

$$\rho^+D^+\frac{d\omega}{d\xi} + Re\,Sc\,(1/w_0 - \omega) = 0. \quad (18)$$

The boundary conditions in dimensionless notation are:

$$\left.\begin{aligned} \phi = 0,\ \theta = \theta_0,\ \omega = 1 \quad &\text{at } \xi = 0, \text{and} \\ \phi = 1,\ \theta = 1,\ \omega = \omega_1 \quad &\text{at } \xi = 1. \end{aligned}\right\}(19)$$

Note, however, that the concentration w_0 at the wall is not known and has to be calculated. The connection between the blowing velocity at the wall v_0 and the concentration of the radiating gas injected at the wall is given by equation (14).

Equations (15), (16), and (18) with boundary conditions (19) compose a system to be solved for the dependent variables ϕ, θ and ω. Since the system of equations is classified as a "two point-boundary-value-problem," and because the integrodifferential equation of energy is nonlinear a closed form solution does not appear possible. Two numerical methods of solution are feasible: (1) forward integrations of conservation equations, and (2) integrations by successive approximations. To use the second method, the differential and integrodifferential equations were converted to integral equations.

Constant property solution

Considerable insight into the heat-transfer process is obtained if it is assumed that the injected absorbing–emitting gas has physical properties not markedly different from the main stream thereby permitting the assumption of constant physical properties. This hypothesis also includes the simplification that the dimensionless heat-transfer results are dependent on a minimum number of parameters, but still retain many of the essential features of the related boundary-layer problem.

Integration and use of the boundary conditions, equation (19), yields the solution of equation (15) in the form:

$$\phi = (e^{Re\,\xi} - 1)/(e^{Re} - 1) \tag{20}$$

Substituting this into the energy equation results in:

$$\frac{d\theta}{d\xi} = Pe[-e_w^* + \theta + Bo\,\Phi + \tfrac{1}{2}E(1-\theta_0) \times (e^{2Re\,\xi} - 1)/(e^{Re} - 1)^2] \tag{21}$$

Integrating equation (21) once with respect to ξ from 0 to ξ and making use of the boundary conditions (19), yields a nonlinear integral equation for the temperature distribution:

$$\begin{aligned}\theta = {} & \theta_0 + (1-\theta_0)\,\xi + Pe\left[\int_0^\xi \theta(t)\,dt - \xi\int_0^1 \theta(t)\,dt\right] \\ & + \frac{2Pe\,Bo}{\tau_o}\{\epsilon_0\theta_0^4\,G(\xi) + [\tfrac{1}{3}\xi + (1-\xi)E_4(\tau_o) \\ & - E_4[\tau_o(1-\xi)] + 2(1-\epsilon_0)E_3(\tau_o)G(\xi)]\,F_d^* \\ & + \tfrac{1}{2}\mu_o^2\left[\xi + e^{-\tau_o/\mu_o}\left(1 - \xi - e^{\tau_o\xi/\mu_o} + \frac{2}{\mu_o}(1-\epsilon_0)\,G(\xi)\right)\right]F_p^* \\ & - \tau_o\int_0^1 \theta^4(t)\,[E_3(\tau_o|\xi - t|) - (1-\xi)E_3(\tau_o t) - \xi E_3[\tau_o(1-t)] \\ & - 2(1-\epsilon_0)E_2(\tau_o t)\,G(\xi)]\,dt\} + \tfrac{1}{4}E\,Pr(1-\theta_0) \\ & [\xi(1-e^{2Re}) - (1-e^{2Re\xi})]/(e^{Re}-1)^2\end{aligned} \tag{22}$$

where

$$G(\xi) = \tfrac{1}{3}(1-\xi) - E_4(\tau_o\xi) + \xi E_4(\tau_o) \tag{23}$$

Variable property solution

The purpose of this section is to present a more realistic analysis of mass-transfer cooling of a surface against incident radiation and to show that the procedure of the previous section may be extended to gases having temperature and concentration dependent properties, while at the same time keeping the number of independent parameters to the minimum. The intent here is not an accurate reproduction of data for physical and optical properties of the gases by analytical expressions, but only a qualitatively correct functional form which represents the gross trends of the properties.

To accomplish this we consider an "ideal gas", which we will define as one for which the following relations hold:

$$\left.\begin{aligned} &\mu^+ = \theta^{\frac{3}{4}}; \quad k^+ = \theta^{\frac{3}{4}}; \quad D^+ = \theta^{\frac{3}{4}}; \\ &\rho = p/RT; \quad c_p = \text{const}; \\ &Pr = \text{const}; \quad Sc = \text{const}.\end{aligned}\right\} \tag{24}$$

These expressions represent approximately the qualitative dependence of the transport properties of an ideal gas such as air.

The absorption coefficient depends on wavelength, gas composition, density and temperature. The assumptions of grayness has eliminated the wavelength. This involves some sort of wavelength average. There are, however, basic difficulties associated with an averaging over wavelength [9]. No distinction is made here between the different mean absorption coefficients, and the justification is that, while these may differ considerably from each other, the variation of either over the temperature range of interest is not dominant. Of the remaining variables, the density is assumed to be implicitly accounted for by the variation with temperature. Thus, as an approximation it is assumed that the mean absorption coefficeint of a mixture of gases composed of non-radiating free-stream gas and the radiating secondary injected species can be expressed by the following relation:

$$\kappa^+ = \omega^\gamma \theta^\alpha \, e^{-\beta(1-\theta)/\theta} \tag{25}$$

where α, β, and γ are appropriate constants. This expression has a qualitatively correct functional form which represents the gross features of the absorption process for such gases as CO_2, H_2O, CH_4, NH_4 and others.

Integrating the momentum equation (15) and species equation (18) once with respect to ξ from 0 to ξ and evaluating the integration constants from the boundary conditions (19), we obtain:

$$\phi = Re\int_0^\xi (\phi/\mu^+)\,d\xi + [1 - Re\int_0^1 (\phi/\mu^+)\,d\xi]\int_0^\xi (d\xi/\mu^+)/(\int_0^1 d\xi/\mu^+), \tag{26}$$

and

$$\omega = 1 - Re\,Sc\int_0^\xi \left(\frac{1}{\rho^+ D^+}\right)\left(\frac{1}{w_0} - \omega\right) d\xi. \tag{27}$$

The non-linear integrodifferential equation (16) can be transformed to a non-linear equation by integrating once with respect to ξ from 0 to ξ. If the integration constants are evaluated by applying the boundary conditions (19), the resulting equation becomes:

$$\theta = \theta_0 + (1-\theta_0)f(\xi) + Pe\left[\int_0^\xi (\theta/k^+)\,d\xi - f(\xi)\int_0^1 (\theta/k^+)\,d\xi\right]$$
$$+ \frac{2Pe\,Bo\,\tau_o}{\tau_1}\left[\int_0^{\zeta(\xi)} (g/k^+\kappa^+)\,d\eta - f(\xi)\int_0^1 (g/k^+\kappa^+)\,d\eta\right.$$
$$\left. - \int_0^1 (\theta^4 h/k^+\kappa^+)\,d\eta\right] + E\,Pe(1-\theta_0)\left[\int_0^\xi (s/k^+)\,d\xi - f(\xi)\int_0^1 (s/k^+)\,d\xi\right], \tag{28}$$

where

$$f(\xi) = \int_0^\xi (d\xi/k^+)/\int_0^1 (d\xi/k^+),$$
$$\zeta(\xi) = \int_0^\xi \kappa^+\,d\xi/\int_0^1 \kappa^+\,d\xi,$$
$$g(\eta) = \chi_0 E_3(\tau_o\eta) - F_d^* E_3[\tau_o(1-\eta)] - \tfrac{1}{2}\mu_o F_p^*\,e^{-\tau_o(1-\eta)/\mu_o},$$
$$\chi_0 = \epsilon_0\theta_0^4 + 2(1-\epsilon_0)[F_d^* E_3(\tau_o) + \tfrac{1}{2}\mu_o F_p^* e^{-\tau_o/\mu} + \tau_o\int_0^1 \theta^4(\eta)E_2(\tau_o\eta)\,d\eta], \tag{29}$$
$$h(\xi,\zeta,\eta) = E_3(\tau_o|\zeta-\eta|) - f(\xi)\{E_3[\tau_o(1-\eta)] - E_3(\tau_o\eta)\} - E_3(\tau_o\eta),$$

and

$$s(\xi) = [\phi(\xi)/2 + \tau_w^*/Re]\,\phi(\xi).$$

Since the temperature depends on the velocity field and indirectly through the absorption coefficient on the concentration, the system of equations (26–28) must be solved simultaneously in order to determine the temperature uniquely. The method of solution is discussed in the following paragraphs.

Method of solution

Successive approximations were carried out on an IBM 7094 digital computer according to the following procedure. The temperature profiles were expressed in terms of a power series, for example:

$$\theta(\xi) = \sum_{i=0}^{k} a_i \xi^i \quad \text{and} \quad \theta^4(\xi) = \sum_{i=0}^{k} b_i \xi^i. \tag{30}$$

This allowed for an exact evaluation of integrals in equation (22). Least squares fits were performed on each calculated temperature profile and an iteration procedure was carried out until the temperature profile converged to within a prescribed limit. Normally the solution to the corresponding non-radiating gas problem was used as the initial guess in the iteration procedure. A similar procedure was followed to find solutions to the variable properties problem. The expressions for the dimensionless absorption coefficient κ^+ and temperature were approximated by polynomials and least square fits were performed for each iteration. The temperature distribution was solved by a forward integration of equation (21) using the Runge–Kutta method. The maximum residual error in each fit was printed out with each iteration so that some estimation of the accuracy of solution could be made. For example, the maximum residual error in approximating θ^4 by a sixth degree polynomial was of the order of 10^{-3} to 10^{-4}.

The rate of convergence depended upon the choice of parameters. For small values of the optical thickness, it was possible to use the method of direct substitution. In more severe cases where the optical thickness was large or where the incident radiation flux was large it was necessary to use an averaging process to compute an adjusted value of the temperature profile for the next iteration. In several cases where more than five iterations were needed in the averaging process, a special procedure [10] was used to speed convergence.

Heat transfer

The heat flux to the stationary wall ($y = 0$) is given by:

$$q = \left[-k\frac{\mathrm{d}T}{\mathrm{d}y} + \mathscr{F}\right]_{y=0} \tag{31}$$

Thus, once the temperature distribution has been determined, the heat transfer can be calculated. The heat flux in dimensionless notation becomes:

$$\begin{aligned} q^* = q/(k_1 T_1/\delta) &= q_c^* + \mathscr{F}^* \\ &= -(k^+\,\mathrm{d}\theta/\mathrm{d}\xi)_{\xi=0} + Bo\,Pe\,\epsilon_0\,\{\theta_0^4 - 2[F_d^* E_3(\tau_o) + \tfrac{1}{2}\mu_o F_p^*\,\mathrm{e}^{-\tau_o/\mu_o} \\ &\qquad + \tau_o \int_0^1 \theta^4(\eta)\,E_2(\tau_o\eta)\,\mathrm{d}\eta]\} \end{aligned} \tag{32}$$

Injection of a non-radiating gas into the layer diminishes the total heat-transfer rate to the surface by reducing the convective part of the flux as has been shown in [8]. In order to determine whether or not a net saving in heat transfer to the surface results from injection of an absorbing–emitting gas a comparison will be made with the heat-transfer rate with injection of a non-radiating gas.

If the gas injected into the layer is non-radiating the total heat transfer to the surface in dimensionless form is

$$q^*_{n-r} = -\left(k^+ \frac{\mathrm{d}\theta_{n-r}}{\mathrm{d}\xi}\right)_{\xi=0} + Bo\, Pe\, \epsilon_0(\theta_0^4 - F_d^* - \mu_o F_p^*) \tag{33}$$

The temperature distribution θ_{n-r} for the case of Couette flow with mass injection of a non-radiating gas having constant physical properties has been given, for example, by Eckert and Schneider [8]. The convective heat transfer to the wall can be expressed as:

$$q_c^* = -\frac{(1-\theta_0)Pe}{e^{Pe}-1}\left\{1 - \frac{E}{2(2-Pr)(e^{Re}-1)^2}\left[Pr(e^{2Re}-1) - 2(e^{Pe}-1)\right]\right\} \tag{34}$$

RESULTS

Independent parameters

Before discussing the results it is desirable to review the parameters that enter into the problem. An inspection of equation (22) reveals that the dimensionless temperature, θ, is a function of ten parameters: $\theta = \theta(Re, Pr, E, \theta_0, \tau_o, Bo, F_d^*, F_p^*, \mu_o, \epsilon_0, \xi)$. For the variable property problem several additional parameters arise. It was considered impractical to cover the full range of variation (which is of physical interest) for each parameter. Results are reported here primarily for the cases when radiation interacts strongly with the gas.

We note that for the case of constant absorption coefficient, the optical thickness of the layer τ_o is given by $\tau_o = \kappa_1\delta = \tau_1$; however, when the absorption coefficient varies with temperature the optical thickness τ_o is related to τ_1 by the equation

$$\tau_o = \int_0^\delta \kappa(y)\,\mathrm{d}y = \tau_1 \int_0^1 \kappa^+(\xi)\,\mathrm{d}\xi \tag{35}$$

and thus is not known *a priori*.† The correspondence between τ_o and τ_1 or τ and ξ is established only after the temperature distribution has been determined (or assumed). For this reason τ_1 and not τ_o is chosen as an independent parameter for the latter case. The physical nature of the results can be understood better when we note that the parameter $Pe\, Bo/\tau_1 = \sigma T_1^3/k_1\kappa_1 = 1/(4N)$ represents the relative magnitude of heat transfer by radiation to that by conduction. The value $N = \infty$ corresponds to pure convection and $N = 0$ to pure radiation. The remaining parameters are familiar ones and need not be discussed here.

† Note that κ_1 corresponds to the absorption coefficient of the injected gas evaluated at the upper wall temperature and a concentration of unity.

Constant properties

Figure 2 presents the variation of the temperature distribution with the emissivity of the stationary wall for dissipationless Couette flow with an intermediate blowing rate parameter, $Re = 2{\cdot}0$. For the purpose of comparison, the temperature distribution for the case when the injected gas is non-radiating is also included in the figure. The temperature profiles for the intermediate values of emissivity, $0 < \epsilon_0 < 1$, fall between the perfectly black, $\epsilon_0 = 1$, and the perfectly reflecting, $\epsilon_0 = 0$, limiting cases and have not been included for the sake of clarity. It is seen from the figure that in shielding a surface from incident radiation by injection of an absorbing–emitting gas, the convective heat transfer is increased at the cool stationary wall. Absorption of radiation produces an effective heat source which in turn increases the temperature gradient and therefore the convective heat transfer at the cool wall. It is also seen that as the surface is made more reflecting the temperature level is increased; with decreasing emissivity more energy is reflected from the stationary wall and absorbed in the layer. Thus, the reduction in radiant heat transfer by absorption in the layer tends to be offset by the resulting increase in convective heat transfer.

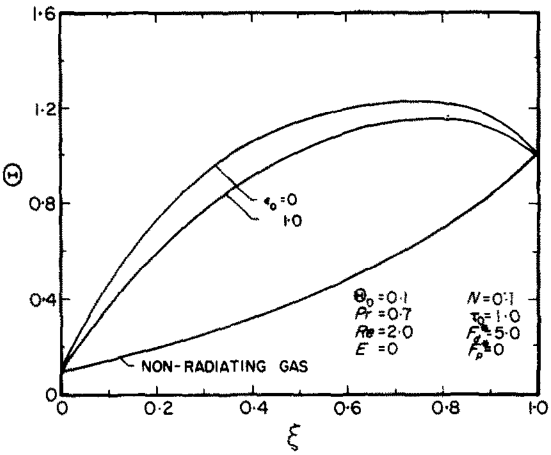

FIG. 2. Effect of emissivity on temperature distribution for dissipationless flow.

The effect of the emissivity on the temperature distribution for a similar case of Couette flow but with viscous dissipation is shown in Fig. 3.

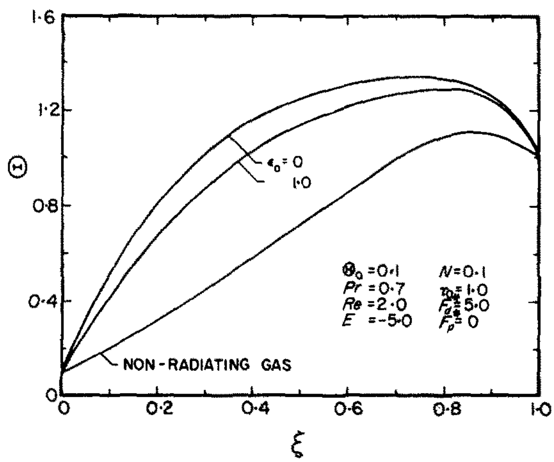

FIG. 3. Effect of emissivity on temperature distribution for flow with viscous dissipation.

The particular value of the Eckert number selected ($E = -5{\cdot}0$) corresponds approximately to Mach number of 4. As expected, a comparison of temperature profiles in Figs. 2 and 3 shows that the temperature level and the gradient are

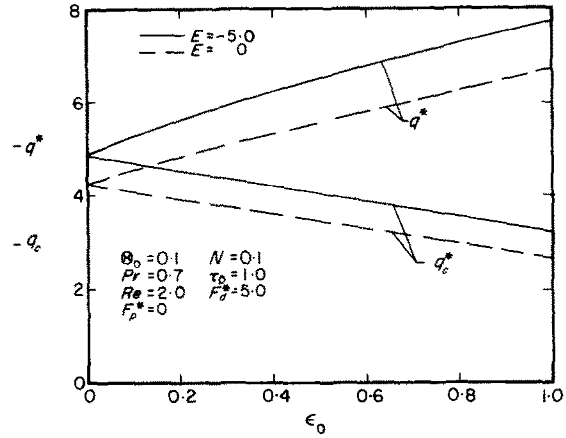

FIG. 4. Comparison of total and convective heat transfer for flow with and without viscous dissipation.

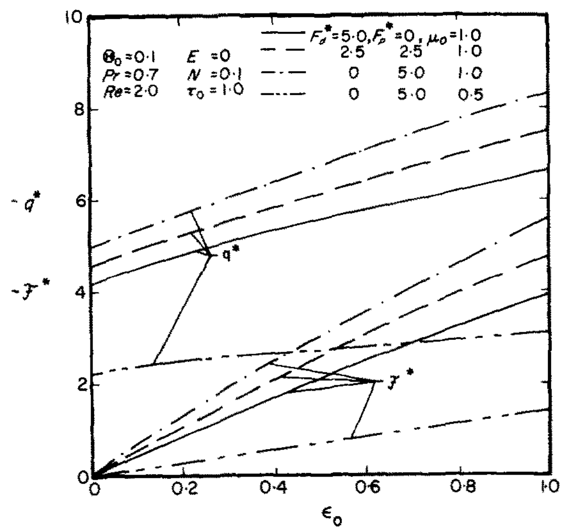

FIG. 5. Effect of incident radiation flux on total and radiative heat transfer.

smaller for the dissipationless case. The effects of the conduction–radiation interaction parameter N, the mass injection parameter Re, and the optical thickness τ_o have been discussed by

Novotny *et al.* [4] and need not be repeated here.

The variation of the total and the convective heat-transfer rates with emissivity for flow with and without viscous dissipation is shown in Fig. 4. It is seen from the figure that the total heat-transfer rates decrease and the convective heat-transfer rates increase almost linearly with decreasing emissivity. The larger values of q^* and q_c^* for the flow with dissipation is expected.

The differences in the total and radiative heat-transfer rates resulting from different types of radiative fluxes incident on the transparent moving plate are illustrated in Fig. 5. An inspection of the figure reveals that both the total and the radiative heat-transfer rates are largest when a parallel beam of radiation falls normally ($\mu_o = 1$) on the transparent plate and smallest when the beam makes an angle of $\theta_0 = \cos^{-1} 0{\cdot}5 = 60°$ with the normal to the plates. These trends are expected. For example, the direct radiation reaching the stationary plate for $\tau_o = 1$ is $2E_3(\tau_o)F_d^* = 0{\cdot}2194\ F_d^*$ for the diffuse flux, $\mu_o \exp(-\tau_o/\mu_o)\,F_p^* = 0{\cdot}3679\ F_p^*$ and $0{\cdot}0677\ F_p^*$ for a parallel flux incident normally and at an angle of 60°, respectively. In the limiting case as $\theta_0 \to \pi/2(\mu_o \to 0)$, the direct radiation flux incident on the stationary plate vanishes as can be readily seen from equation (17).

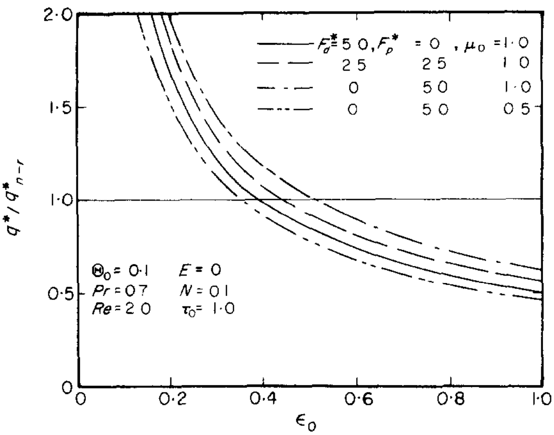

FIG. 6. Ratio of heat transfer with injected radiating gas to that with injected non-radiating gas.

A comparison of heat-transfer rate with absorption of radiation in the layer to that without absorption for a given injection rate is shown in Fig. 6. Examination of the figure reveals that for all cases involving black surfaces, it is advantageous to inject an absorbing gas. However, the effectiveness of the gas in shielding the surface from an incident radiation flux diminishes with decreasing emissivity. For small emissivities it is not desirable to inject an absorbing gas to shield the surface from radiation because of increase in convective heat transfer.

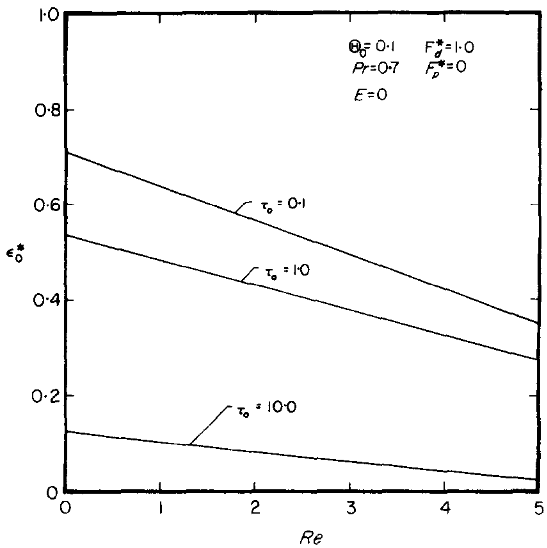

FIG. 7. Variation of the break even emissivity as a function of the blowing Reynolds number for $N/\tau_o = 0{\cdot}1$.

The emissivity for which the heat transfer rate is the same whether the injected gas is radiating or not is called the break even emissivity [1] and is denoted by ϵ_0^*. For emissivities below the break even value, the non-radiating gas would provide better heat transfer protection than a radiating gas. Inspection of Figs. 5 and 6 reveals that ϵ_0^* decreases with decreasing radiative flux.

The break even emissivities determined in a a similar manner for other values of the mass injection parameter Re and three values of optical thickness τ_o are shown in Fig. 7. It is seen that ϵ_0^* decreases almost linearly with Re and is smallest for the largest optical thickness ($\tau_o =$ 10) considered. For surface emissivities above lines corresponding to a given optical thickness τ_o, the heat protection is best afforded by injection of a radiating gas, and below the curves by injection of transparent gas. Similar trends in ϵ_0^* with Re have been reported by Howe [1] who considered the shielding of stagnation surfaces against incident radiation by transpiration of an absorbing but non-emitting gas. Additional calculations not presented here have shown that for given values of optical thickness and incident flux the break even emissivities are rather insensitive to the conduction–radiation interaction parameter N and the Eckert number E.

Variable properties

A comparison of the temperature distributions between the constant and variable property cases (for selected values of α, β and γ) is given in Fig. 8. Because of the particular dependence of the thermal conductivity on temperature the trends of θ are not unexpected. Again as in the case of constant properties, a decrease in the emissivity increases the temperature gradient at the stationary wall; however, the convective heat-transfer rate is larger for the case of constant thermal conductivity as can be seen from Fig. 9. This is due to the fact that the thermal conductivity at the cool wall is higher in the latter case. For the particular values of parameters chosen, the convective heat-transfer rate

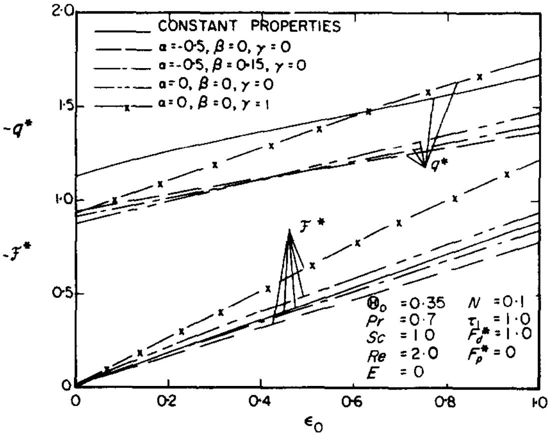

FIG. 9. Comparison of total and radiant heat-transfer rates as a function of emissivity for constant and variable property cases.

is approximately equal to the radiative heat-transfer rate (at $\epsilon_0 = 1$), and $\mathscr{F}^*$ is seen to be affected little by the variation of thermal conductivity and absorption coefficient with the temperature.

Examination of Fig. 9 reveals that for the cases where the injected and the free stream gases have the same physical properties, i.e. $\gamma = 0$, the total heat transfer is largest for the constant property case. A comparison of results for which τ_o is the same shows that the increase in temperature level caused by the variable thermal conductivity is associated with an increase in the radiation flux. Further examination of the figure indicates that the radiation flux is smallest for the case when the absorption coefficient decreases most rapidly with increasing temperature. The optical thickness, however, becomes larger in this case, i.e. $\tau_o \simeq 1{\cdot}2$ for $\alpha = -0{\cdot}5$, $\beta = \gamma = 0$ at $\epsilon_0 = 1{\cdot}0$. For fixed τ_o, it is expected that the most effective shielding would

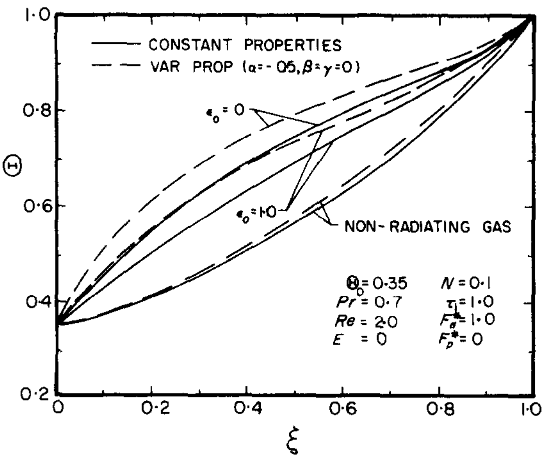

FIG. 8. Comparison of temperature distributions for constant and variable property cases.

be obtained with a gas having a large absorption coefficient in the high temperature region away from the stationary wall. The calculations for the case when the absorption coefficient depends on the concentration of the injected gas were based on a value of $w_1 = 0$ for the purpose of

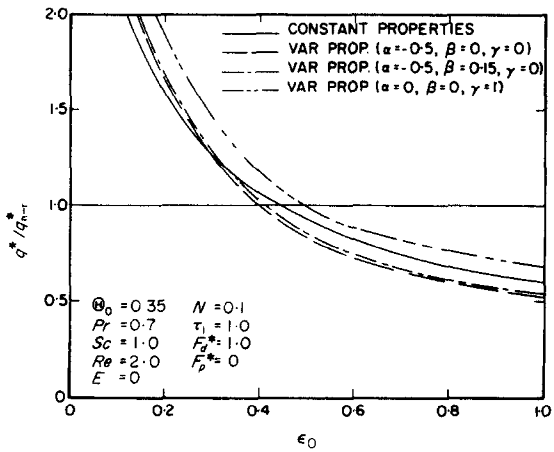

FIG. 10. Ratio of heat transfer with injected radiating gas to that with injected non-radiating gas for constant and variable property cases.

simulating boundary-layer flow. The large increase in radiation and total heat-transfer rates are primarily due to the decrease in the optical thickness to $\tau_o \simeq 0{\cdot}65$.

The results of Fig. 10 show that the break even emissivity is affected little by the variation of the absorption coefficient and the thermal conductivity with temperature. It is expected that for larger incident fluxes and optical thicknesses these effects would be more pronounced.

CONCLUSIONS

The results of the analysis of Couette flow model shows that the total heat transfer to the surface is always reduced by injection of an absorbing–emitting gas into the layer if the stationary wall is black. This protection diminishes to zero as the emissivity decreases to the break even condition. For the emissivity below the break even condition it is disadvantageous to inject a radiating gas, and a transparent gas offers better protection against incident thermal radiation. It is concluded that for a given optical thickness the gas which has the largest absorption coefficient in the high temperature region is the most effective in shielding the cool surface.

The break even emissivity depends most strongly on the radiation flux at the wall and the optical thickness of the layer τ_o. Transpiration of a radiating gas is most advantageous for large incident radiation fluxes where the surfaces are nearly black. With increasing optical thickness the effectiveness of the radiating gas in shielding a surface against incident radiation is extended to lower values of emissivity.

ACKNOWLEDGEMENT

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Résumé—L'écoulement laminaire de Couette a été analysé afin de montrer les effets de l'injection d'un gaz absorbant et émettant un rayonnement sur l'interaction entre la vonvection et le rayonnement, sur les profils de température, et sur le transport de chaleur à la paroi immobile. Les cas des propriétés physiques constantes et variables sont considérés tous les deux. Les résultats du transport de chaleur sont présentés et l'on montre que la réduction du transport de chaleur par rayonnement à la surface due à l'injection d'un gaz rayonnant est accompagnée par une augmentation du transport de chaleur par convection. On trouve que l'efficacité du gaz rayonnant comme écran pour la surface par rapport à un flux de rayonnement incident diminue lorsque l'émissivité décroît. On détermine la condition d'émissivité intermédiaire entre les extrêmes d'une surface noire et d'une surface parfaitement réfléchissante au-dessus de laquelle il est avantageux d'injecter un gaz absorbant.

Zusammenfassung—Die laminare Couetteströmung wurde analysiert um zu zeigen, welchen Einfluss die Einblasung eines strahlungsabsorbierenden—emittierenden Gases auf die Wechselwirkung von Konvektion mit Strahlung, auf die Temperaturprofile und auf den Wärmeübergang an der fasten Oberfläche besitzt. Es wird sowohl der Fall konstanter als auch variabler Stoffeigenschaften betrachtet. Die Ergebnisse des Wärmeüberganges sind angegeben und es wird gezeigt, dass die Abnahme im Wärmeübergang durch Strahlung beim Einblasen eines strahlenden Gases von einer Zunahme des konvektiven Wärmeüberganges begleitet wird. Es ergibt sich, dass die Wirksamkeit des strahlenden Gases für die Abschirmung der Oberfläche von einfallender Strahlung abnimmt mkt abnehmender Emissivität. Die Emissionsbedingung zwischen den Extremen der schwarzen und der vollständig reflektierenden Oberfläche für die eine Einblasung des absorbierenden Gases vorteilhaft ist, wird bestimmt.

Аннотация—На анализе ламинарного потока Куэтта показано влияние вдува поглощающего излучение и излучающего газа на конвекцию, температурные профили, а также перенос тепла к стационарной поверхности. Рассматривается случай с постоянными, а также и с переменными физическими свойствами. Приводятся данные по теплообмену и показано, что снижение скорости радиационной теплопередачи на поверхности путем вдува излучающего газа сопровождается возрастанием скорости конвективного теплообмена. Нашли, что эффективность излучающего газа в защите поверхности от падающего потока радиации снижается с понижением излучательной способности. Определены условия излучения между предельными случаями черной и абсолютно отражающей поверхности, над которыми целесообразно вдувать абсорбирующий газ.